American University of Beirut
Department of Computer Science
CMPS 211 – Discrete Mathematics – Fall 15/16
Assignment 4 Solution

Please solve the following exercises and submit **BEFORE 5:00 pm of Wednesday 21st of October.** Submit on Moodle.

Exercise 1 (10 points)

Let A, B, and C be sets. Show that:

a) $(A \cup B) \subseteq (A \cup B \cup C)$
$\{x | x \in (A \cup B)\}$ (Assumption)
$\Rightarrow \{x | x \in A \vee x \in B\}$
$\Rightarrow \{x | x \in A \vee x \in B \vee x \in C\}$(Addition)
$\Rightarrow \{x | x \in (A \cup B \cup C)\}$

b) $(A \cap B \cap C) \subseteq (A \cap B)$
$\{x | x \in (A \cap B \cap C)\}$ (Assumption)
$\Rightarrow \{x | x \in A \wedge x \in B \wedge x \in C\}$ (Definition of intersection)
$\Rightarrow \{x | x \in A \wedge x \in B\}$(Simplification)
$\Rightarrow \{x | x \in (A \cap B)\}$

c) $(A - B) - C \subseteq A - C$
$\{x | x \in (A - B) - C\}$(Assumption)
$\Rightarrow \{x | x \in A \wedge x \notin B \wedge x \notin C\}$ (Definition of difference)
$\Rightarrow \{x | x \in A \wedge x \notin C\}$ (Simplification)
$\Rightarrow \{x | x \in (A - C)\}$

d) $(A - C) \cap (C - B) = \emptyset$
$(A \cap C) \cap (B - C)$
$= (A \cap C) \cap (B \cap \bar{C})$
$= (A \cap B) \cap (C \cap \bar{C})$
$= (A \cap B) \cap \phi$ (Complement law)
$= \phi$ (Domination law)

e) $(B - A) \cup (C - A) = (B \cup C) - A$
$\{x | x \in (B - A) \vee x \in (C - A)\}$(Assumption)
$\Rightarrow \{x | (x \in B \wedge x \notin A) \vee (x \in C \wedge x \notin A)\}$ (Definition of difference)
$\Rightarrow \{x | x \notin A \wedge (x \in B \vee x \in C)\}$ anti – Distribution
$\Rightarrow \{x | x \in (B \cup C) - A\}$(Definition of difference)

Exercise 2 (10 points)

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a) (A ∪ (B - C)) ∩ D
b) (A−B)∪(A−C)∪(B−C)

Exercise 3 (10 points)

Can you conclude that A = B if A, B, and C are sets such that

a) A∪C = B∪C?
No,
Counter example:
Let A={1,2,3,4,5}, B={1,2,3,4}, C={5}
AUC = {1,2,3,4,5}
BUC = {1,2,3,4,5}
We get, AUC=BUC but A does not equal B.

b) A∩C = B∩C?
No,
Counter example:
Let A={1,2,3}
B={1,2}
C={1}
A∩C={1}= B∩C but A does not equal B.

c) A-C = B-C and C-A = B-A?
No,
Let A= {1,2,3}, B={1,4,3,6}, C={2,4,6}
A-C=B-C= {1,3}
C-A={4,6}= B-A
But A and B are not equal.

Exercise 4 (10 points)

Find the domain and range of these functions. Note that in each case in order to find the domain, determine the set of elements assigned values by the function.

a) The function that assigns to each nonnegative integer its first digit
b) The function that assigns to a bit string the number of one bits in the string
c) The function that assigns to each non-negative integer its 3rd power and returns the last digit
d) The function that raises 2 to the non-negative integer assigned to the function

Exercise 5 (10 points)

Give an example of a function from **Z** to **N** that is

a) one-to-one but not onto.
$F(x) = x^2$

b) onto but not one-to-one
$F(x) = \lfloor |x|/3 \rfloor$

c) both onto and one-to-one (but different from the identity function)
$$F(x) = \begin{cases} 2(x-1) & \text{if } x > 0 \\ -2x+1 & \text{if } x < 0 \\ x & \text{if } x = 0 \end{cases}$$

d) neither one-to-one nor onto.
$F(x) = |x|+1$

Exercise 6 (10 points)

Determine whether each of these functions is a bijection from **R** to **R**.

a) $f(x) = -3x+1$
b) $f(x) = -3x^2-7$
c) $f(x) = (x+1)/(x-2)$
d) $f(x) = x^3+1$

Exercise 7 (10 points)

Let $f(x) = ax^2 + bx + c$ and $g(x) = 2dx + e$, where a, b, c, d and e are constants. Determine necessary and sufficient conditions on the constants a, b, c, d and e so that $f \circ g = g \circ f$.

$$\begin{aligned} f(g(x)) &= a(2dx + e)^2 + b(2dx + e) + c \\ &= 4ad^2x^2 + 4adex + ae^2 + 2bdx + be + c \\ &= (4ad^2)x^2 + (4ade + 2bd)x + ae^2 + be + c \end{aligned}$$

$$\begin{aligned} g(f(x)) &= 2d(ax^2 + bx + c) + e \\ &= (2ad)x^2 + (2bd)x + 2dc + e \end{aligned}$$

So it is the following conditions are necessary:
$4ad^2 = 2ad$
$\rightarrow 2d = 1$

→ d = ½

$2bd = 4ade + 2bd$
→ 4ade = 0,
→ a = 0 or e = 0

$ae^2 + be + c = 2dc + e$
→ $ae^2 + be + c = 2(\frac{1}{2})c + e$
→ $ae^2 + be = e$
→ e(ae + b) = e
→ ae + b = 1; ae = 0 since a = 0 or e = 0
→then b = 1

Exercise 8 (10 points)

Let f be a function from A to B. Let S and T be subsets of B. Show that:

Let Q and R be 2 subsets in A such that f(Q) → S and f(R) → T
$f^{-1}(S) \rightarrow Q$ and $f^{-1}(T) \rightarrow R$

a) $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

Assme we have x such that $x \in f^{-1}(S \cup T)$
==> $x = f^{-1}(o)$ for some $o \in (S \cup T)$
==> $o \in S \vee o \in T$
==> $f^{-1}(o) \in Q \vee f^{-1}(o) \in R$
==> $x \in Q \vee \in R$
==> $x \in (Q \cup R)$

Assme we have x such that $x \in (f^{-1}(S) \cup f^{-1}(T))$
==> $x \in (Q \cup R)$
Then they are equivalent

b) $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$.

Exercise 9 (10 points)

Prove or disprove each of these statements about the floor and ceiling functions.

a) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for all real numbers x.
b) $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.
c) $\lceil \lceil x/3 \rceil /2 \rceil = \lceil x/6 \rceil$ all real numbers x.

d) $\lfloor\sqrt{\lceil x\rceil}\rfloor = \lfloor\sqrt{x}\rfloor$ for all positive real numbers x.
e) $\lfloor x\rfloor+\lfloor 2y\rfloor+\lfloor 2x+y\rfloor\leq\lfloor 3x\rfloor+\lfloor 3y\rfloor$ for all real numbers x and y .

Exercise 10 (10 points)

Fuzzy sets are used in artificial intelligence. Each element in the universal set U has a **degree of membership**, which is a real number between 0 and 1 (including 0 and 1), in a fuzzy set S.The fuzzy set S is denoted by listing the elements with their degrees of membership (elements with 0 degree of membership are not listed).
For instance, we write {0.6 Alice, 0.9 Brian, 0.4 Fred, 0.1 Oscar, 0.5 Rita} for the set F (of famous people) to indicate that Alice has a 0.6 degree of membership in F, Brian has a 0.9 degree of membership in F, Fred has a 0.4 degree of membership in F, Oscar has a 0.1 degree of membership in F, and Rita has a 0.5 degree of membership in F (so that Brian is the most famous and Oscar is the least famous of these people). Also suppose that R is the set of rich people with R = {0.4 Alice, 0.8 Brian, 0.2 Fred, 0.9 Oscar, 0.7 Rita}.

a) The **complement** of a fuzzy set S is the set S, with the degree of the membership of an element in S equal to 1 minus the degree of membership of this element in S.
Find F (the fuzzy set of people who are not famous) and R (the fuzzy set of people who are not rich).
b) The **union** of two fuzzy sets S and T is the fuzzy set $S \cup T$, where the degree of membership of an element in $S \cup T$ is the maximum of the degrees of membership of this element in S and in T .
Find the fuzzy set $F \cup R$ of rich or famous people.
c) The **intersection** of two fuzzy sets S and T is the fuzzy set $S \cap T$, where the degree of membership of an element in $S \cap T$ is the minimum of the degrees of membership of this element in S and in T .
Find the fuzzy set $F \cap R$ of rich and famous people.